SPACE-TIME PICTURE OF THE STRING FRAGMENTATION AND THE FUSION OF COLOUR STRINGS

V.V. Vechernin

St.-Petersburg State University
E-mail: vechernin@pobox.spbu.ru

Abstract

It is shown that naïve two stage scenario of the soft multiparticle production in hadronic and nuclear collisions at high energy, when at first stage the colour strings are formed and at the second stage these strings, or some other (higher colour) strings formed due to fusion of primary strings, are decaying, emitting observed particles, encounters some difficulties at the attempt to analyse the space-time picture of the process. Simple analysis shows the dominant is the process when the formation and the decay of a string occur in parallel - a string breaks into two parts already at rather small length (about 1÷2 fm in its c.m. system), then the process repeats in the pieces and so on. Nevertheless it is proved to be possible to agree the string fusion idea with the space-time picture of a string decay. In the framework of the Artru-Mennessier model of a string fragmentation the simple interpretation of the homogeneity of the rapidity distribution for hadrons produced from the decay of a single string at high energy is presented and the analytical estimate for the density of this rapidity distribution is obtained.

1 Introduction. AMOR model of string fragmentation

Soft and semihard parts of the multiparticle production at high energy are successfully described in terms of colour strings stretched between the projectile and target [1, 2] in the framework of a two-stage scenario, when at the first stage a certain number of colour strings stretched between the incoming partons are formed and at the second stage these strings decay into the observed secondary hadrons. In the case of nuclear collision, the number of strings grows with the growing energy and atomic numbers of colliding nuclei, and one has to take into account the interaction between strings in the form of their fusion and/or percolation [3]-[8] (see Fig.1). The aim of the present paper is to analyse to what extent the two-stage scenario is compatible with the space-time picture of the process.

We'll consider the space-time evolution of a string in the framework of the classical approach with the action

$$I = -\gamma \int \sqrt{(\dot{x}x')^2 - \dot{x}^2 x'^2}\; d\sigma d\tau \ . \tag{1}$$

We'll also restrict our consideration to the simplest case of so-called "yo-yo" string. As is well known (see, for example, [9, 10]) in this case the motion of the string is the oscillations, the half-cycle of which is shown as a rectangle $OACB$ on the space-time diagram in Fig.2.

The 4-momentum P of the string is connected with the diagonal vector c of this rectangle: $P = \gamma c$ and the mass of the string M is given by $M^2 = P^2 = \gamma^2 c^2$. One can decomposes diagonal vector c on the sum of two light cone vectors: $c = a + b$; $a^2 = b^2 = 0$, $a_- = b_+ = 0$, where $a_\pm \equiv a_0 \pm a_z$ and $b_\pm \equiv b_0 \pm b_z$. Then we have $M^2 = 2\gamma^2(ab) = \gamma^2 a_+ b_- = 2\gamma^2|OA||OB| =$

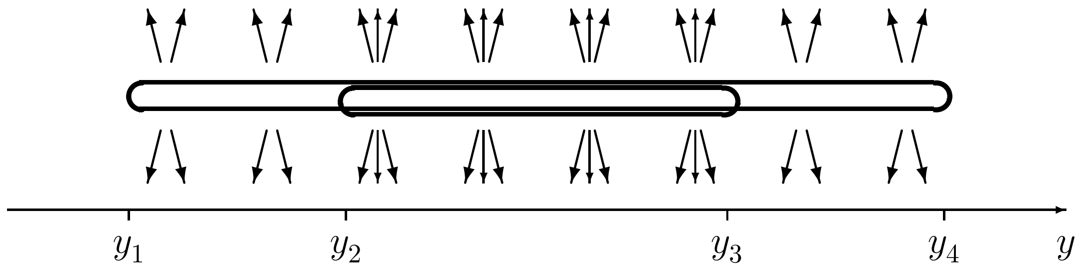

Figure 1: The string overlap in rapidity (y) in two stage scenario.

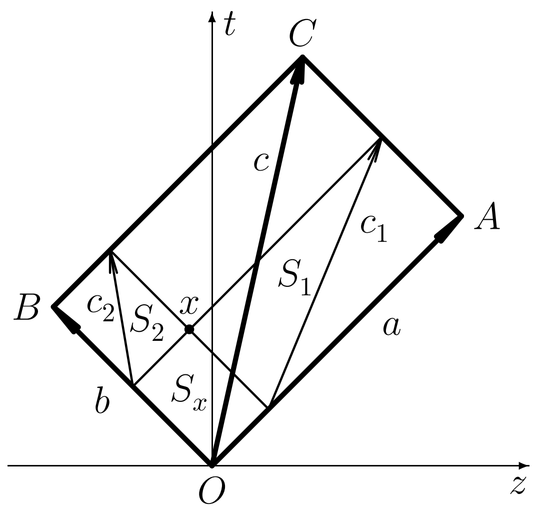

Figure 2: The string decay in the AMOR model.

$2\gamma^2 S_E(OACB)$, as $a_+ = a_0 + a_z = 2a_0 = \sqrt{2}|OA|$ and $b_- = b_0 - b_z = 2b_0 = \sqrt{2}|OB|$. Here $S_E(OACB)$ is the Euclidean area of the rectangle $OACB$.

At first sight it seems a little bit strange that the squared mass of the string is proportional to Euclidean area of the $OACB$, because the meaning of the action I is the area sweeping by the string in Minkowski space. To clarify this point note, that the element of the area, formed by two arbitrary vectors $a = (a_0, \mathbf{a})$ and $b = (b_0, \mathbf{b})$ in Minkowski space S_M is given by

$$S_M^2 = (ab)^2 - a^2 b^2 = S^2 - \Delta \ ,$$

where $(ab) = (a_0 b_0 - \mathbf{ab})$, $a^2 = a_0^2 - \mathbf{a^2}$, $b^2 = b_0^2 - \mathbf{b^2}$ and $S^2 = (a_0 \mathbf{b} - b_0 \mathbf{a})^2$, $\Delta = \mathbf{a^2 b^2} - (\mathbf{ab})^\mathbf{2}$. The element of the area in Euclidean space S_E is given by

$$S_E^2 = (a \times b)^2 = a^2 b^2 - (ab)^2 = S^2 + \Delta \ ,$$

where now $(ab) = (a_0 b_0 + \mathbf{ab})$, $a^2 = a_0^2 + \mathbf{a^2}$, $b^2 = b_0^2 + \mathbf{b^2}$ and the S^2 and the Δ are the same. In the case of yo-yo string $\Delta = a_z^2 b_z^2 - (a_z b_z)^2 = 0$ and hence $S_M = S_E = S$. So for yo-yo string we have

$$P = \gamma c \ , \qquad M^2 = 2\gamma^2 S = 2\gamma^2 S_E(OACB) \tag{2}$$

After the split of the string in the space-time point x two strings with the momenta $P_1 = \gamma c_1$, $P_2 = \gamma c_2$ and the masses $M_1^2 = 2\gamma^2 S_1$, $M_2^2 = 2\gamma^2 S_2$ are formed (see Fig.2). In principle any chain of rectangles connected by the corners and going from the point A to B corresponds to some possible decay of the initial string to substrings. At that the substrings with small area, of order of particle masses, are associated with produced particles.

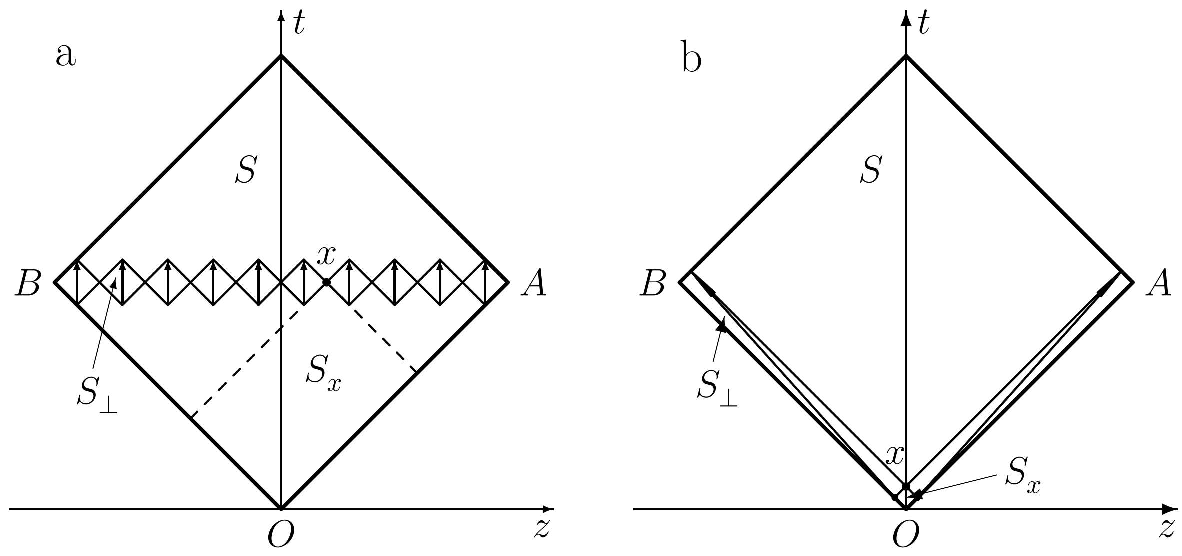

Figure 3: Two examples of the rare string decays.

In Fig.3 we present two specific examples of the string decay. In Fig.3a all decay points x are at the same time, when the length of the string is maximal in its c.m. system. In this case all rapidities of produced particles are equal to zero. In Fig.3b the decay point x is very close to the origin O, so that the area of two formed rectangles are of order of particle masses. In this case the string decays on two particles with the minimal and maximal possible values of rapidity. Both cases do not correspond to the typical physical situation, when one has more or less homogeneous distribution of produced particles in rapidity. The reason is the small probability of events in Fig.3. The dominant process is shown in Fig.4 (see below).

We'll consider the string fragmentation in the framework of the Artru-Mennessier AMOR model [11, 12], which is used in the VENUS event generator [10] and has in our opinion more fundamental physical foundations, than the Lund model [13], used for example in PYTHIA (see discussion in [10, 13]). In the AMOR model the probability of the string split in the space-time point x is proportional to the probability of the absence of splitting points in the area S_x (see Fig.2):

$$dP(x) = S_0^{-1}\,[1 - P(x)]\,dS_x\ , \tag{3}$$

which (by analogy with an unstable particle decay) leads to

$$P(x) = 1 - \exp\left(-S_x/S_0\right)\ , \qquad dP(x) = S_0^{-1}\,\exp\left(-S_x/S_0\right)dS_x\ , \qquad \langle S_x\rangle = S_0\ . \tag{4}$$

2 Analytical estimate of the rapidity distribution

Let us now estimate the value of involving parameters. The the string tension parameter γ in (1) is connected with the slope α' of Regge trajectories: $\gamma^{-1} = 2\pi\alpha'$ [9]. For α'=0.9 GeV^{-2} we have γ=0.18 GeV^2 (c=$\hbar$=1). From the parameters of the potential connecting heavy quarks in nonrelativistic models one obtains the close value γ=0.19 GeV^2. So we take

$$\gamma = 0.18\,GeV^2 = 4.6 fm^{-2} = (0.47 fm)^{-2}\ . \tag{5}$$

The parameter S_0, specifying the string decay probability in the AMOR model, can be expressed through the dimensionless so-called 'area law parameter' α_0 of the VENUS event

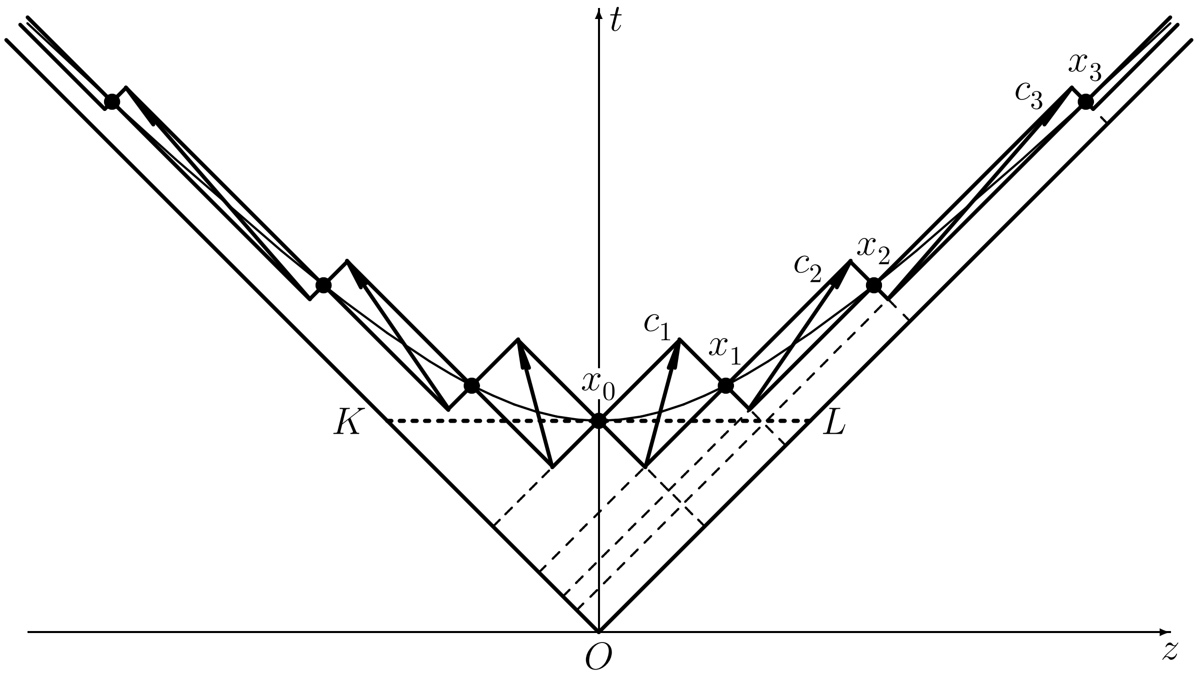

Figure 4: The dominant string decay.

generator: $S_0^{-1} = 2\alpha_0\gamma$. From the comparison of the VENUS event generator results with the experimental data one finds in [10] α_0=0.6. So we have

$$S_0 = \langle S_x \rangle = (2\alpha_0\gamma)^{-1} = 4.6\,GeV^{-2} = 0.18 fm^2 = (0.43 fm)^2\ . \qquad (6)$$

We introduce also the parameter $S_{0\perp}$=$\langle S_\perp \rangle$ - the mean area at which the string is associated with a produced particle (see Fig.3). By (2) $S_{0\perp} = m_{0\perp}^2/(2\gamma^2)$, where $m_{0\perp}^2 \equiv \langle m^2 + p_\perp^2 \rangle$ - the average transverse mass of produced particles. In this way one can effectively take into account the transverse momentum of produced particles in the framework of yo-yo string model. So we have

$$S_{0\perp} = \langle S_\perp \rangle = m_{0\perp}^2/(2\gamma^2) = \langle m^2 + p_\perp^2 \rangle/(2\gamma^2)\ . \qquad (7)$$

The typical values of parameters $m_{0\perp}^2$ and $S_{0\perp}$ for different particles are presented below in the Table.

	$m_{0\perp}^2$, GeV^2	$S_{0\perp}$, fm^2	β	dN/dy
π	0.11	0.07	0.4	1.5
ρ	0.6	0.36	2.0	0.75
N	1.0	0.6	3.3	0.63

From the (6) and the Table we see that values of S_0 and $S_{0\perp}$ are of the same order of magnitude: $S_0 = (0.43 fm)^2$ and $S_{0\perp} = (0.26 \div 0.78 fm)^2$. At that the total area S of the rectangle $OACB$, corresponding to the initial string (see Fig.3), is much larger at high energies ($S \gg S_0,\ S_{0\perp}$). For example, for a string with the mass $M^2 = (100\,GeV)^2$ we find $S = (78 fm)^2$ and $|AB| = 110 fm$.

Recalling formula (4) we understand now the reason of the small probability of the process in Fig.3a. In this case $S_x \sim S \gg S_0 = \langle S_x \rangle$ and the exponent in (4) is small. For the process in Fig.3b we have $S_x \ll S_\perp \sim S_{0\perp} \sim S_0 = \langle S_x \rangle \ll S$ and the probability of this process is small due to small phase volume ($S_x \ll \langle S_x \rangle$, see (4)). Clear that the dominant processes will be the ones with $S_x \sim \langle S_x \rangle = S_0$ (see Fig.4).

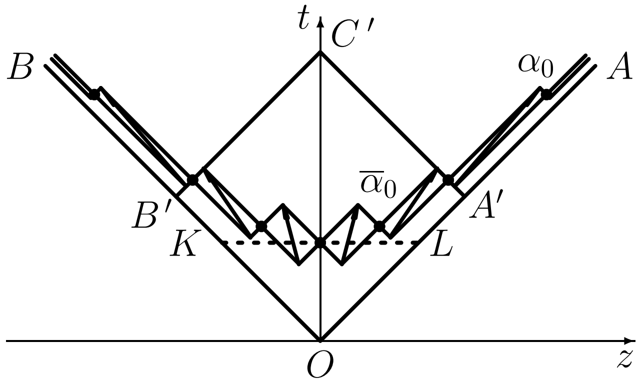

Figure 5: The space-time picture of the string fusion.

For rough estimate of the rapidity distribution of produced particles in this case we'll consider that S_x is equal for all splitting points: $S_{x_i} = S_0$ (see Figs.3 and 4). This leads to the condition

$$2S_{x_i} = x_i^2 = t_i^2 - z_i^2 = 2S_0 \ , \tag{8}$$

which means that all string splitting points $x_i = (t_i, z_i)$ are situated on the hyperbola (8).

We'll also suppose for rough estimate that the transverse masses of all produced particles is also equal. By (2) this leads to the condition $S_{i\perp} = S_{0\perp}$, which gives (see Figs.3 and 4):

$$2S_{i\perp} = c_i^2 = 2S_{0\perp} = m_{0\perp}^2/\gamma^2 \ . \tag{9}$$

For estimate we'll also consider that the first split of a string occurs in its middle, as this situation of the point $x_0 = (t_0, z_0)$ on the segment KL corresponds to the maximal value of S_{x_0} (see Fig.4). Note that the length of a string (in its c.m. system) at the moment of the first split (t_0) is equal to

$$|KL| = 2z_0 = 2\sqrt{2S_0} = 1.2\, fm \ . \tag{10}$$

After that the condition (9) fixes uniquely the positions of all break points $x_i = (t_i, z_i)$ on the hyperbola (8). Then one can calculate all diagonal vectors c_i in Fig.4 and find by (2) the momenta ($p_i = \gamma c_i$) and rapidities (y_i) of the produced particles. As a result one finds

$$y_i = (1/2)\ln(p_{i+}/p_{i+}) = (1/2)\ln(c_{i+}/c_{i+}) = (i - 1/2)F(\beta) \ , \tag{11}$$

$$F(\beta) = \ln\left[\ 1 + \beta/2 + \sqrt{\beta(1+\beta/4)}\ \right] \ , \tag{12}$$

$$\beta = S_{0\perp}/S_0 = \alpha_0 m_{0\perp}^2/\gamma = \alpha_0 \langle m^2 + p_\perp^2\rangle/\gamma \ , \tag{13}$$

where we used (6) and (7). From (11) we see that the produced particles are homogeneously distributed in rapidity and the density of this rapidity distribution is equal to

$$dN/dy = 1/F(\beta) \ . \tag{14}$$

The numerical estimates by formulae (12)-(14) are presented above for different values of the transverse mass parameter $m_{0\perp}^2$ in two last columns of the Table. One can see that we obtain the reasonable values for such rough estimate. Note that we have supposed that the particles only of one sort can be produced. This leads to 1.5 particles per unit of rapidity for pions (including charged and neutral pions), 0.75 for ρ mesons (which gives again 1.5 for pions) and 0.63 for nucleons. In reality, when different particles can be produced in the decay of the string, the reasonable value of charged particles produced from the decay of one string per unit of rapidity is about 1.1 (see, for example, estimates in [8]).

3 Conclusion

In conclusion we would like to discuss: is the string fusion picture shown in Fig.1 compatible with the dominant space-time picture of string fragmentation shown in Fig.4? The answer is positive. One must only always keep in mind that the picture of the string fusion in Fig.1 concerns the lengths of strings in rapidity space. The corresponding space-time picture are shown in Fig.5. In this figure overlapping of two strings $OA'C'B'$ and $OACB$ with different masses (different lengths in rapidity) is shown.

To take into account the string fusion one has to use in the region $OA'C'B'$ the higher value $\overline{\alpha}_0$ of the area law parameter (6), describing the string fragmentation process, and the usual value α_0 outside this domain, in the rest of the region $OACB$. From formulae (12)-(14) we see that the higher value of α_0 (the lower value of S_0 (6)) leads to the lower density of the produced particles rapidity distribution.

We see also in Fig.5 that the formation and the decay of the fused string occur in parallel and the string breaks into two parts already at rather small length ($|KL|$ is about $1\div2$ fm in the string c.m. system). This is compatible with the string fusion picture in rapidity space shown in Fig.1, as by (11) the particles with rapidities in the region (y_2, y_3) in Fig.1 are being produced from the domain $OA'C'B'$ in Fig.5 with higher value of the area law parameter α_0.

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